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## Deformation Effects on Microwave Reflectivity

J. A. NEMES AND H. N. JONES

*Structural Integrity Branch  
Material Science and Technology Division*

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# DEFORMATION EFFECTS ON MICROWAVE REFLECTIVITY

## INTRODUCTION

The interaction of the mechanical and electromagnetic behavior of materials has been observed in several important areas in engineering and applied physics. Included among these are examples in the areas of piezoelectricity and photoelasticity. Both of these subjects were first systematically treated in the nineteenth century. Voigt<sup>1</sup> (1890's) and Neumann<sup>2</sup> (1840's) respectively are generally credited with the development of the theory for each of these fields. A complete description of the field of photoelasticity, which is related to the work discussed in this paper, is given in ref. 3.

The photoelastic and piezoelectric effects can be treated within the broader field of the response of elastic-dielectric materials. A comprehensive treatment of these materials, formulated in the context of nonlinear continuum mechanics is given by Toupin<sup>4</sup>. In his development, piezoelectricity, photoelasticity, and other physical phenomena related to coupled mechanical and electromagnetic effects are treated within a unified theory.

The interaction of electromagnetic waves with organic materials or organic matrix composites, which are subject to deformations from body or surface forces, requires an understanding of the coupling between mechanical and electromagnetic response. This type of coupled effect has been studied in detail by Adkins and Rivlin<sup>5</sup>. The authors consider the propagation of electromagnetic waves in a dielectric material held in a fixed state of deformation and, in particular, a state of pure torsion. Electromagnetic wave propagation is found to be dependent on the dielectric properties of the material and the magnitude of the deformation. The present paper investigates whether such an effect is found to exist in dielectric materials when exposed to electromagnetic radiation at microwave frequencies. In particular, it is desired to determine if the reflectivity of these materials is modified by varying the state of deformation. In addition, if the reflectivity is altered by the deformation, could such observations be characterized in the context of a generalized development including coupled mechanical-electromagnetic behavior?

## DEVELOPMENT OF THE THEORY

In this section a general development of electromagnetic wave propagation in anisotropic media is presented. Such a description is required to study the effect that an arbitrary deformation has on material reflectivity. If the propagation of electromagnetic waves is a function of deformation, then an arbitrary deformation would transform a material, which is initially isotropic, to one which is anisotropic.

The propagation of plane electromagnetic waves in a homogeneous material is governed by Maxwell's equations<sup>6</sup>. These are given in rationalized units as

$$\nabla \cdot \bar{D} = \rho \quad (1)$$

$$\nabla \cdot \bar{B} = 0 \quad (2)$$

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$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \quad (3)$$

$$\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J} \quad (4)$$

where,

$$\begin{aligned} \vec{D} &= \text{electric displacement} \\ \vec{B} &= \text{magnetic induction} \\ \vec{E} &= \text{electric field intensity} \\ \vec{H} &= \text{magnetic field intensity} \\ \rho &= \text{free charge density} \\ \vec{J} &= \text{current density.} \end{aligned}$$

The above equations are in themselves indeterminate, and must be supplemented by constitutive relations. For an isotropic material which responds independently of deformation, these constitutive relations may be stated as

$$\vec{J} = \sigma \vec{E} \quad (5)$$

$$\vec{B} = \mu \vec{H} \quad (6)$$

$$\vec{D} = \epsilon \vec{E} \quad (7)$$

where  $\sigma, \mu, \epsilon$  are constants designated as the electric conductivity, magnetic permeability, and dielectric permittivity, respectively. Alternately, if the material is anisotropic, the quantities relating the vector fields would be second order tensors as defined by

$$\vec{J} = \boldsymbol{\sigma} \vec{E} \quad (5a)$$

$$\vec{B} = \boldsymbol{\mu} \vec{H} \quad (6a)$$

$$\vec{D} = \boldsymbol{\epsilon} \vec{E}. \quad (7a)$$

In some cases a material may be taken to be isotropic magnetically but not electrically. In such a case, as will be considered here, the appropriate constitutive relations are eqs. 5a, 6, and 7a.

Taking the curl of eq. 3 and making use of the constitutive relations expressed by eqs. 5a, 6, and 7a leads to the expression

$$\nabla \times (\nabla \times \vec{E}) + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu \sigma \frac{\partial \vec{E}}{\partial t} = 0. \quad (8)$$

Then making use of the identity  $\nabla \times (\nabla \times \vec{E}) = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$  and assuming the absence of free charges, eq. 8 becomes

$$\nabla^2 \vec{E} - \nabla (\nabla \cdot \vec{E}) = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu \sigma \frac{\partial \vec{E}}{\partial t} \quad (9)$$

which forms a set of three coupled damped wave equations. Note that in the isotropic case, where  $\vec{D}$  is parallel to  $\vec{E}$ , the second term of eq. 9 is zero because of eq. 1. If it is further assumed that the vector fields are plane harmonic such that

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad (10)$$

and likewise for the other quantities, then eq. 9 can be written as

$$-k^2 \vec{E} + \vec{k}(\vec{k} \cdot \vec{E}) = -\omega^2 \mu \epsilon \vec{E} - i\omega \mu \sigma \vec{E} \quad (11)$$

where  $\vec{k}$  is the propagation vector. Further, by introducing a complex dielectric tensor defined as

$$\epsilon^* = \epsilon - \frac{i}{\omega} \sigma \quad (12)$$

eq. (11) reduces to

$$k^2 \vec{E} - \vec{k}(\vec{k} \cdot \vec{E}) = \omega^2 \mu \epsilon^* \vec{E}. \quad (13)$$

Equation (13) can then be used to determine the wave propagation velocities within the medium of interest.

An investigation into deformation effects on electromagnetic response can be made by considering  $\epsilon$  and  $\sigma$  to be functions of the infinitesimal strain tensor  $\mathbf{e}$  ( $\mu$  could also be considered a function of strain, if desired). Thus,

$$\epsilon = \epsilon(\mathbf{e}) \quad (14)$$

$$\sigma = \sigma(\mathbf{e}).$$

A general representation for eq. 14 in component form is given as

$$\epsilon_{ij} = \alpha_0 \delta_{ij} + \alpha_1 e_{ij} + \alpha_2 e_{ik} e_{kj} \quad (15)$$

$$\sigma_{ij} = \beta_0 \delta_{ij} + \beta_1 e_{ij} + \beta_2 e_{ik} e_{kj} \quad (16)$$

where  $e_{ij}$  are the components of the infinitesimal strain tensor.  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$  are undetermined coefficients, which are polynomial functions of the three invariants of  $\mathbf{e}$ , given by

$$e_{ii}, \quad \frac{1}{2}(e_{ii} e_{jj} - e_{ij} e_{ji}), \quad |e_{ij}|. \quad (17)$$

## EXPERIMENTS

In order to determine the coefficients in eqs. 15 and 16, and thus assess the dependence of the electromagnetic response on deformation for a given material, experiments must be performed. Briefly, the experiment selected for this purpose consisted of the measurement of the reflectance of a rectangular panel subjected to microwave radiation in both a natural state and in a state of uniaxial stress. The incident electric field is linearly polarized and propagated at normal incidence to the panel. There are several advantages to performing such an experiment, which are discussed. First,



the solution of eq. 13 for all but the simplest geometries is extremely difficult. Therefore, the use of a thin slab provides a geometry for which an exact solution can readily be determined. Secondly, determination of the deformation field can be accomplished for a uniaxially stressed slab at least in the elastic regime. In such a field, the principal stress and therefore the principal strain directions correspond to the principal directions of the plate, namely the height, width, and thickness directions. In addition to simplifying the strain tensor, the use of the principal directions is important for another reason. Consideration of eqs. 15 and 16 shows that for a material with an imposed strain tensor such that  $e_{ij} = 0$  for  $i \neq j$ ,  $\epsilon$  and  $\sigma$  will also be in the principal coordinate system. Since the polarization of the incident wave can be aligned with one of the principal directions, there will be a single wave transmitted through the plate. This would not be the case for a state of generalized deformation, which would lead to tensors  $\epsilon$  and  $\sigma$  having non-zero off diagonal components. Under these more general deformations, solution of eq. 13 leads to two possible characteristic waves. Thus, there would be two waves propagated through the material with distinct wave velocities. This result, termed birefringence, is of fundamental importance in photoelasticity, where measurement of the phase difference between the two waves leads to a determination of the stress field in the body. A more complete discussion of wave propagation in anisotropic media can be found in many advanced texts on electrodynamics, such as ref. 7.

In the experiment as configured, reflectivity can then be determined by the components of the dielectric and conductivity tensors appropriate to that direction of polarization only. With the description of  $\epsilon$  and  $\sigma$  given in eqs. 15 and 16, the reflectivity for polarization parallel to the direction of the applied load could differ from the reflectivity for polarization perpendicular to the direction of applied load.

## ANALYTICAL SOLUTION FOR EXPERIMENTAL CONFIGURATION

The problem of determining the reflectivity of a flat panel subjected to a normally incident electromagnetic wave is depicted in Fig. 1. For the specific problem of interest, medium 1 and medium 3 are lossless, isotropic, and equivalent, having properties  $\epsilon^{(1)}$  and  $\mu^{(1)}$ . Medium 2 is anisotropic but is aligned such that the direction of polarization of the incident wave is along the principal axis. Thus, in this coordinate system the only non-zero components of the dielectric tensor are  $\epsilon_{11}^{*(2)}$ ,  $\epsilon_{22}^{*(2)}$ , and  $\epsilon_{33}^{*(2)}$ . The permeability of medium 2 is designated as  $\mu^{(2)}$ . The dielectric components here are taken to be complex in accordance with eq. 12. The propagation constant,  $k^{(1)}$ , for the isotropic media is simply  $\sqrt{\omega^2 \mu^{(1)} \epsilon^{(1)}}$ . However, for medium 2 the propagation constant is determined from eq. 13. Because of continuity conditions at the surface,  $\vec{E}$  is parallel to  $\vec{D}$ , and since  $\nabla \cdot \vec{D}$  is zero by eq. 1, then the second term of eq. 13 will be zero. The resulting expression

$$k^2 \vec{E} = \omega^2 \mu \epsilon^* \vec{E} \quad (18)$$

can be solved for the propagation constant. For a wave polarized in the  $x_1$  direction,  $k_1^{(2)}$  is equal to  $\sqrt{\omega^2 \mu^{(2)} \epsilon_{11}^{*(2)}}$  and for a wave polarized in the  $x_2$  direction,  $k_2^{(2)}$  is  $\sqrt{\omega^2 \mu^{(2)} \epsilon_{22}^{*(2)}}$ .

From Fig. 1, it is seen that medium 1 contains the incident wave along with a reflected wave, which is actually composed of many reflected waves. In medium 2 there is a left travelling wave and a right travelling wave. Medium 3 contains only the right travelling transmitted wave. The reflection coefficient,  $\Gamma$ , defined as the ratio of the reflected electric field intensity to the incident electric field intensity,  $\frac{E_o^r}{E_o^i}$ , is determined by applying the following boundary conditions at each interface

$$\vec{E}^i + \vec{E}^r = \vec{E}^t \quad (19)$$

$$\bar{H}^i + \bar{H}^r = \bar{H}^t. \quad (20)$$

Equation 20 is used in conjunction with Maxwell equation 3 and constitutive relation Eq. 6 to provide a second expression relating  $E_i$ ,  $E_r$ , and  $E_t$ . Thus with these two expressions, the ratio of  $E_r/E_i$  can be obtained. Plonus<sup>7</sup> gives an expression for the reflection coefficient of an isotropic panel of thickness  $d$ . Extending that result to account for the anisotropy of medium 2 yields

$$\Gamma = \frac{\Gamma_{12} + \Gamma_{23} e^{-2ik_i^{(2)}d}}{1 + \Gamma_{12}\Gamma_{23}e^{-2ik_i^{(2)}d}} \quad (21)$$

where,

$$\Gamma_{12} = -\Gamma_{23} = \frac{\sqrt{\mu^{(2)}/\epsilon_{ii}^{*(2)}} - \sqrt{\mu^{(1)}/\epsilon^{(1)}}}{\sqrt{\mu^{(2)}/\epsilon_{ii}^{*(2)}} + \sqrt{\mu^{(1)}/\epsilon^{(1)}}} \quad (\text{no sum on } i)$$

The subscript  $i$  on  $\epsilon_{ii}$  and  $k_i$  denotes the direction that the incident wave is polarized. The reflectivity,  $R$ , is a measure of the reflected power and is determined from the reflection coefficient simply by  $R = |\Gamma|^2$ . The reflectivity is usually expressed in decibels (dB). Thus, if the amplitude of the reflected field is 10% of the incident field, the reflectivity is given by

$$R = 10 \log (.10)^2 = -20 \text{ dB}$$

Under conditions of uniaxial stress, the infinitesimal strain tensor is written in matrix form as

$$[e_{ij}] = \begin{bmatrix} T/E & 0 & 0 \\ 0 & -\nu T/E & 0 \\ 0 & 0 & -\nu T/E \end{bmatrix} \quad (22)$$

where,

$T$  is the applied stress,  
 $E$  is the Young's Modulus, and  
 $\nu$  is Poisson's ratio.

Now if only the first order strain terms are considered in eqs. 15 and 16, the following components of the  $\epsilon$  and  $\sigma$  are obtained:

$$\epsilon_{11} = \alpha_0 + \alpha_1 \frac{T}{E}$$

$$\epsilon_{22} = \alpha_0 - \alpha_1 \frac{\nu T}{E}$$

$$\sigma_{11} = \beta_0 + \beta_1 \frac{T}{E}$$

$$\sigma_{22} = \beta_0 - \beta_1 \frac{\nu T}{E}$$

The thickness of the panel during deformation is



$$d = d_0 \left[ 1 - \frac{\nu T}{E} \right] \quad (23)$$

where  $d_0$  is the original panel thickness.

## MATERIALS

Three different material samples were included in this assessment. Each sample was fabricated into a 12 in. x 12 in. panel to facilitate making the microwave measurements. The first sample was an epoxy of epon 815 cured with versamid 140 (diamine polyimide) and cast to a final thickness of .275 in. The second sample was a fiberglass/epoxy panel which was built up from 15 layers of glass cloth impregnated with epoxy. The resulting thickness was .179 in. The final sample included in the study was a hybrid panel, whose interior was constructed of Epon 815 epoxy with a high volume percentage of glass microballoons. Since the microballoons are essentially air, they have the effect of decreasing the average dielectric constant. This layer is then sandwiched between face sheets of fiberglass/epoxy. Each face sheet consisted of a single layer of the glass cloth. In addition, a thin aluminum foil sheet was bonded to the back face of the panel. The foil served as a ground plane to prevent transmission of the wave through the panel. Thus, it should be noted that the analytical formulation of the thin slab developed in the previous subsection does not apply to this panel, since there is no transmitted wave in medium 3. The resulting thickness of the hybrid panel was 0.165 in..

The mechanical properties of the materials used in the three samples are given below:

epoxy:	$E = .60 \times 10^6$ psi
	$\nu = .35$
fiberglass/epoxy:	$E = 4.0 \times 10^6$ psi
	$\nu = .40$

## MEASUREMENTS

Measurements of reflectivity were made at near normal incidence using the "NRL Arch," which is shown schematically in Fig. 2. The arch permits swept frequency measurements between 2 and 18 GHz. Transmitting and receiving antennas were mounted within the arch at the desired angle. The horns are dually polarized, thereby enabling measurements to be made both parallel and transverse to the arch. These directions correspond to the  $x_2$  and  $x_1$  directions of the panel coordinate system, respectively. Under normal operation a horizontal platform at the center of the arch is used to support the specimen, as shown in Fig. 2.

Application of load to the panels was accomplished by using a specially developed portable load frame. The frame is configured in the horizontal position, such that the panel is at the center of the arch thus eliminating the need for the platform. Due to the large cross-sectional area of the panels and high stiffness of the fiberglass/epoxy panel, a minimum 30,000 lb capacity was required to produce significant deformations. A photograph of the load frame underneath the arch is shown in Fig. 3. Since the load frame is highly reflective, it was necessary to shield all exposed surfaces of the frame, leaving only approximately a 9 in. x 9 in. window where the panel would be subjected to the microwave source. A photograph of the shielded frame is shown in Fig. 4.

The tests that were performed during the evaluation are shown in Table I. For all of the materials, both polarizations were run at each load increment. The epoxy and hybrid specimens were run at the zero load and at approximately 25% of its computed ultimate strength. Premature failure of the specimens at the grips prevented further measurements. The fiberglass/epoxy panel was run at much smaller load increments. Because of the high stiffness of the fiberglass, even the highest load reached during the test was only about 15% of the ultimate. Limitations of the load frame prohibited any tests at higher loads.

## RESULTS AND DISCUSSION

Although tests were conducted on the three materials shown in Table I, only the results for the epoxy will be discussed in any detail. The reason for this is that both the fiberglass/epoxy and the hybrid panel involve multiple constituents, which serve only to complicate interpretation of the results in the context of the theory presented earlier for homogeneous materials. Extension of the theory to heterogeneous material could be performed at a later time.

Figure 5 shows the reflectivity of the epoxy panel prior to application of any load. The differences between parallel and transverse polarizations are essentially negligible. Therefore, only the one curve is shown. The reflectivity as measured while a 6090 lb load was applied is shown in Fig. 6. In this case some differences for the two polarizations are apparent. In Fig. 7 the reflection of the unloaded and loaded panels are superimposed for the case of parallel polarization. As can be seen, the effect of deformation basically involves both a shift of the peak along with a change in amplitude. In order to understand these changes, it is first desirable to identify the source of possible causes. Examination of eq. 21 shows that the reflectivity is dependent upon  $k_i$ ,  $e_{ii}$ , and  $d$ . The effect that change in panel thickness has on reflectivity can be evaluated by considering eq. 23. For an applied stress of 1845 psi, the deformed thickness is computed to be 0.27470 in. Then considering the properties

$$\epsilon_{11}^{(2)} = \epsilon_{22}^{(2)} = 2.35 - j10i$$

$$\mu^{(2)} = 1.0$$

which are approximately those of an epoxy, the change in reflectivity due to thickness change alone is computed from eq. 21. Over the frequency range 6-16 Ghz, the maximum change is computed to be no greater than .2 dB. As can be seen from Fig. 7, this represents a small fraction of the total change in reflectivity.

Thus at least in a macroscopic sense, the change in reflectivity can be discussed by considering both the dielectric and conductivity tensors to be a function of the strain tensor as described by eqs. 15 and 16. Obviously, such a description is not complete without a knowledge of the six functions  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ . Although theoretically these functions could be determined from the type of tests performed, no attempt will be made to quantify them. They will, however be discussed in qualitative terms. First, since the deformations under consideration are small, terms higher than 1st order need not be considered. Thus the relationship can be described with a knowledge of just four functions. Each of these terms are functions of the scalar invariants of the strain tensor. Since Fig. 6 shows that the change in reflectivity during deformation does not differ greatly for the two directions of polarization, then it can be concluded that  $\alpha_1$  and  $\beta_1$  are small at least for infinitesimal strains. Note that if  $\alpha_1$  and  $\beta_1$  were zero then the reflectivity would be the same for both polarizations during deformation. The major portion of the differences depicted in Fig. 7 are described by  $\alpha_0$  and  $\beta_0$ . Examination of the three strain invariants in eq. 17 shows that only the first invariant involves linear

terms. This invariant is interpreted as the dilatation or volume change occurring during deformation. Thus, the changes in reflectivity associated with deformation can be interpreted as being principally due to volume or density change but that some directional aspects are apparent.

In order to obtain some idea of the magnitude of the effect that deformation has on reflectivity, several calculations will be presented. Using eq. 21 with  $d$  equal to the thickness of the epoxy slab, reflectivity is computed over the range of 2-18 GHz for two different values of  $\epsilon'$  which varied by 10 percent. The results are shown in Fig. 8. Similarly, the effect of a 50 percent variation in  $\epsilon''$  on reflectivity is shown in Fig. 9. By using these results in conjunction with the experimental results shown in Fig. 7, rough estimates on the magnitude of change caused by deformation could be obtained.

Results of the reflectivity measurements on the hybrid panel and on the fiberglass/epoxy panel are presented in Figs. 10-13. Without attempting to provide a detailed description of these results, it should be noted that like the epoxy panel, these materials exhibited changes in reflectivity. These changes include both a shift in the peak of minimum reflection and also a change in the amplitude of that minimum. One note should be made concerning the measurements on the fiberglass/epoxy. Since measurements were made at several load increments, it was originally expected that some conclusions could be made to determine if the reflectivity changes were linear in deformation or exhibited some other functional relationship. There are two reasons that such conclusions can not be reached. First, as was previously mentioned, that even though measurements were taken at several load increments, the total range of measurements were all at relatively small percentages of the materials ultimate strength. Thus, large changes in reflectivity would not be expected over such a small range. The second reason concerns the deformation behavior of the fiberglass/epoxy. Since the fiberglass was constructed from a woven cloth, deformation, at least initially, is not linear with applied loading. Such a linear behavior would not be seen until the woven cloth becomes "locked". This may explain why fairly large changes of reflectivity are exhibited during the first load increments, but a relatively small amount is seen with the later increments.

## SUMMARY AND CONCLUSIONS

This paper has presented an evaluation of the effects of mechanical deformation on the reflectivity of dielectric materials subjected to electromagnetic waves over a range of microwave frequencies. Beginning with the premise that the dielectric and conductivity tensors are polynomial functions of the infinitesimal strain tensor, a coupled mechanical-electromagnetic theory was formulated. The effect of a general deformation on electromagnetic response was then included by considering the propagation of waves through an anisotropic medium. The development of an anisotropic theory to the point of determining the wave propagation constants has been included in the paper.

In order to make an assessment of the magnitude of the effect that deformation would have on real materials, experiments on flat slabs were conducted for three different materials. By recording the reflectivity of the panels subjected to a microwave source at normal incidence both prior to and during deformation such an assessment can be made. By orienting the incident polarization along one of the principal axis of deformation, changes in reflectivity were then identified with only three variables-slab thickness and one component each of the dielectric and conductivity tensor.

The results of the tests have been evaluated in light of the theory presented. A qualitative interpretation would suggest that some directional aspects are apparent in the change in components of the dielectric and conductivity tensors, thus necessitating an anisotropic theory. However, the bulk of the change in reflectivity is associated with dilatational changes which are directionally invariant.

Although the work presented in this paper does not quantify the relationship between mechanical deformations and electromagnetic response, the following significant results have been found

1. The reflectivity of common structural materials exposed to microwave radiation was found to depend upon mechanical deformation.
2. The change in reflectivity was found to exhibit both directionally dependent and directionally invariant components.
3. The experimental results could be interpreted qualitatively by using a coupled mechanical-electromagnetic theory, which considers the dielectric and conductivity tensors to be a function of the infinitesimal strain tensor.

The results of this work has shown that in applications for which a precise knowledge of electromagnetic response is required, the state of mechanical deformation, along with other internal variables, such as temperature, must be considered in the formulation of the appropriate constitutive equations.

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Table 1 — Test Matrix

Run No.	Material	Load (lbs.)	Polarization*
1	epoxy	0.	T
2	epoxy	0.	P
3	epoxy	6090.	T
4	epoxy	6090.	P
5	hybrid	0.	T
6	hybrid	0.	P
7	hybrid	4568.	T
8	hybrid	4568.	P
9	F/G	0.	T
10	F/G	0.	P
11	F/G	7613.	T
12	F/G	7613.	P
13	F/G	11420.	T
14	F/G	11420.	P
15	F/G	15226.	T
16	F/G	15226.	P
17	F/G	19033.	T
18	F/G	19033.	P
19	F/G	22840.	T
20	F/G	22840.	P

\*T = Transverse to the arch

P = Parallel with arch



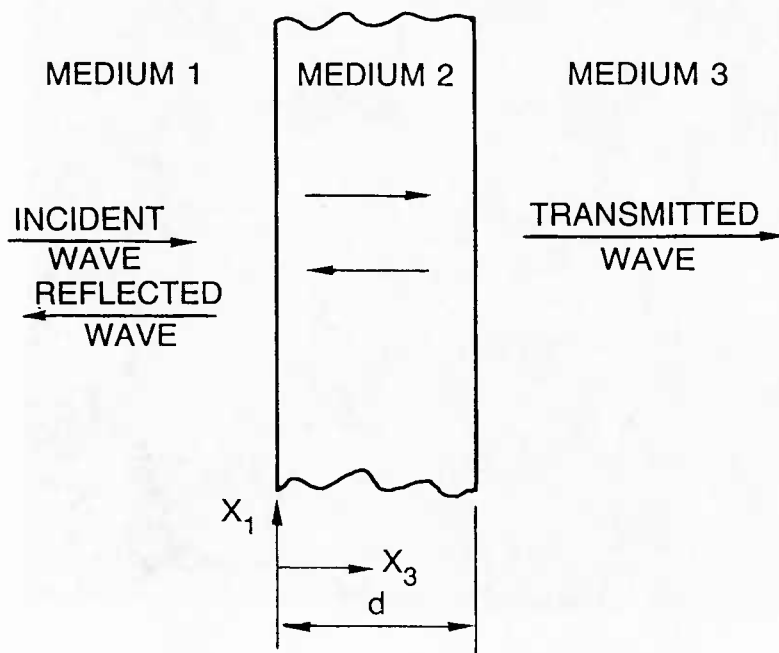


Fig. 1 — Slab subjected to normally incident wave

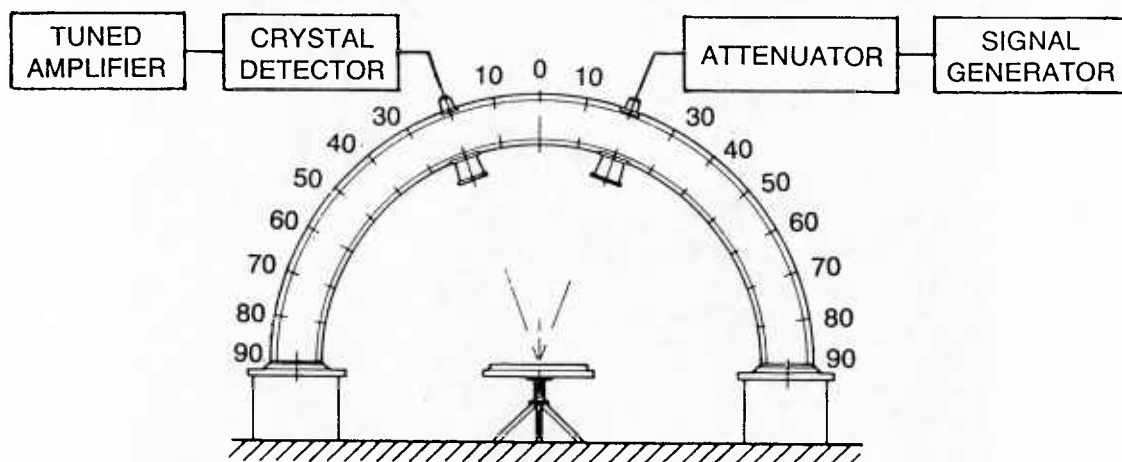


Fig. 2 — NRL arch



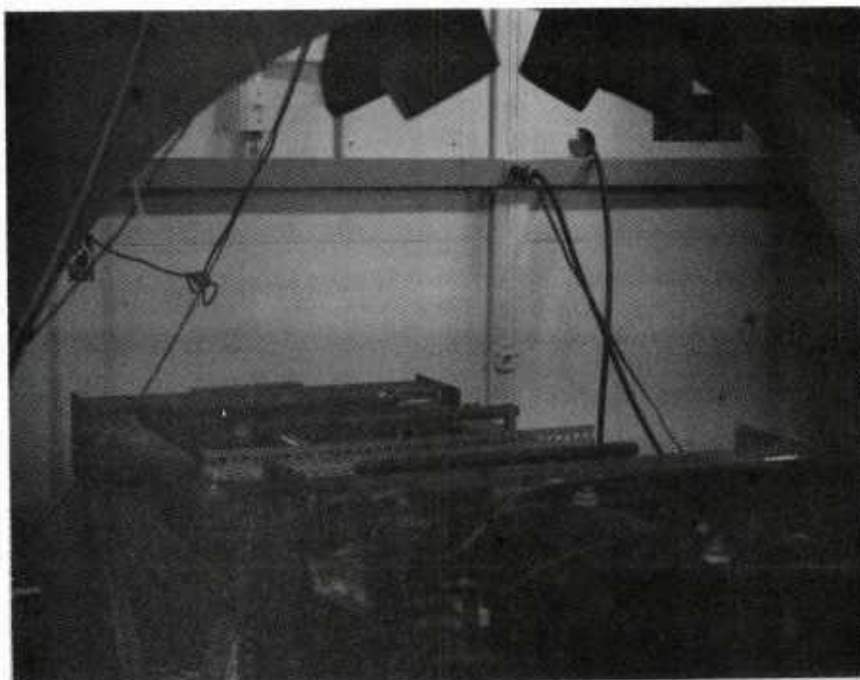


Fig. 3 — Load frame positioned under the arch

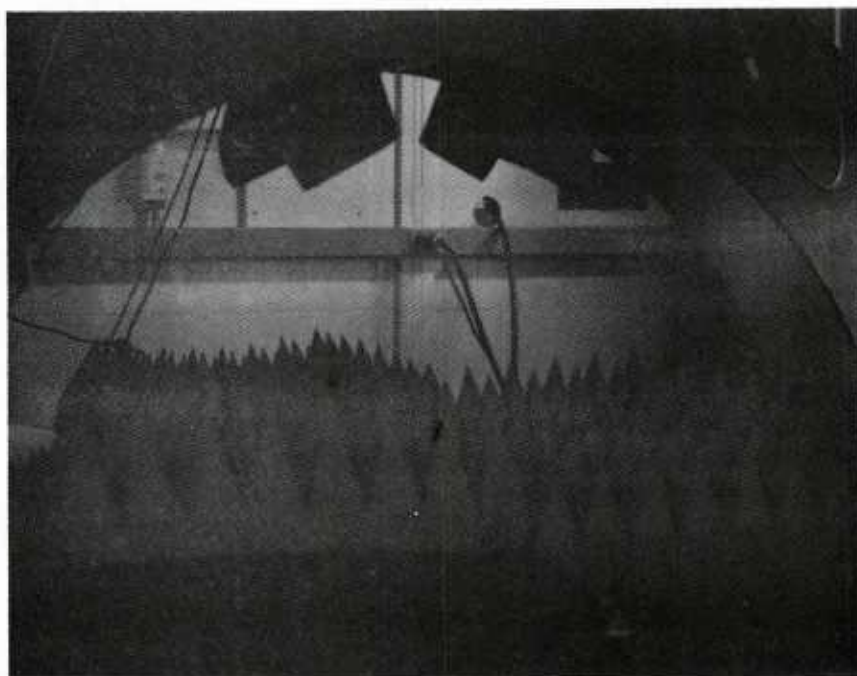


Fig. 4 — Shielding of the load frame

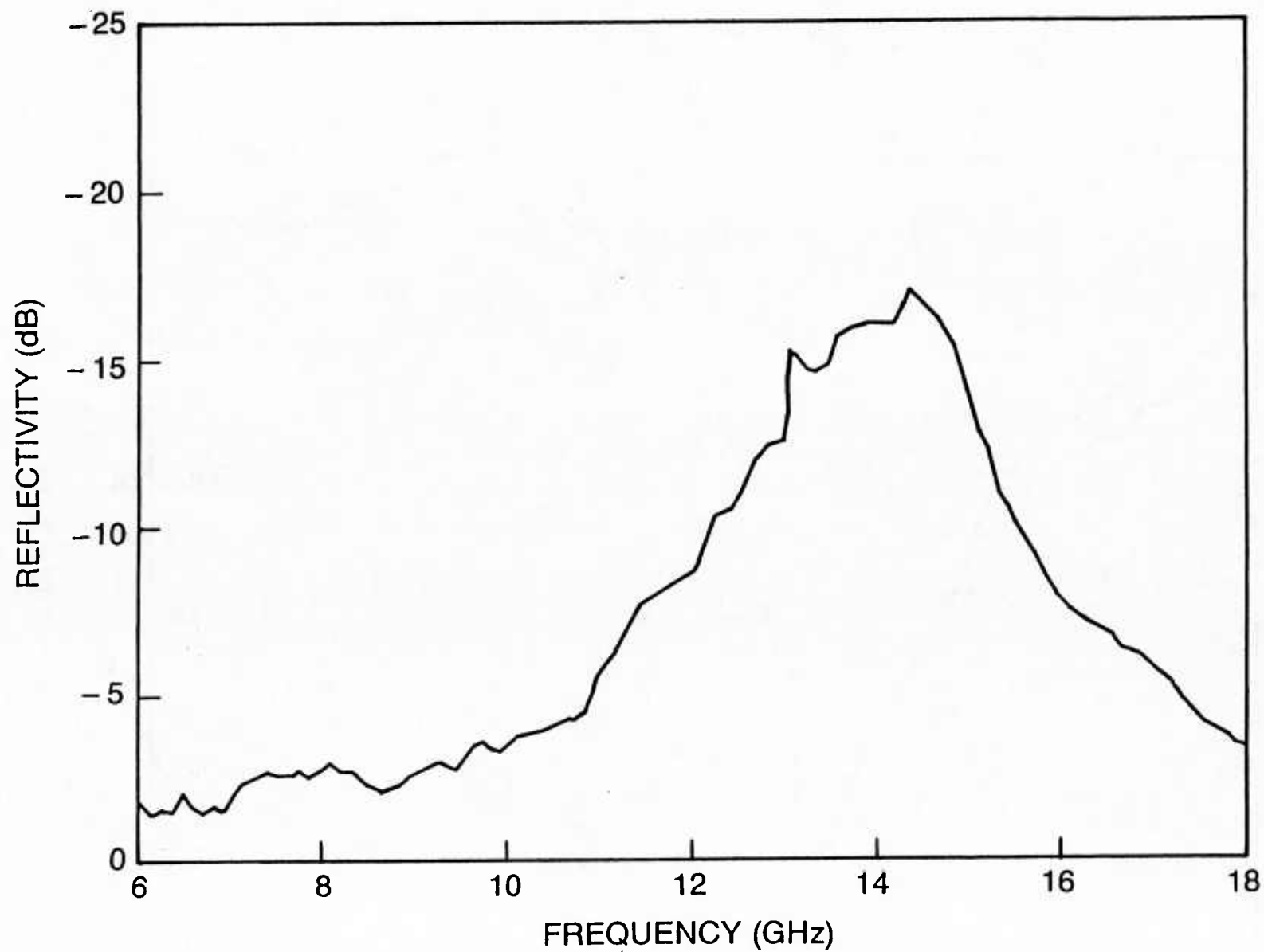


Fig. 5 — Reflectivity of epoxy panel at zero load

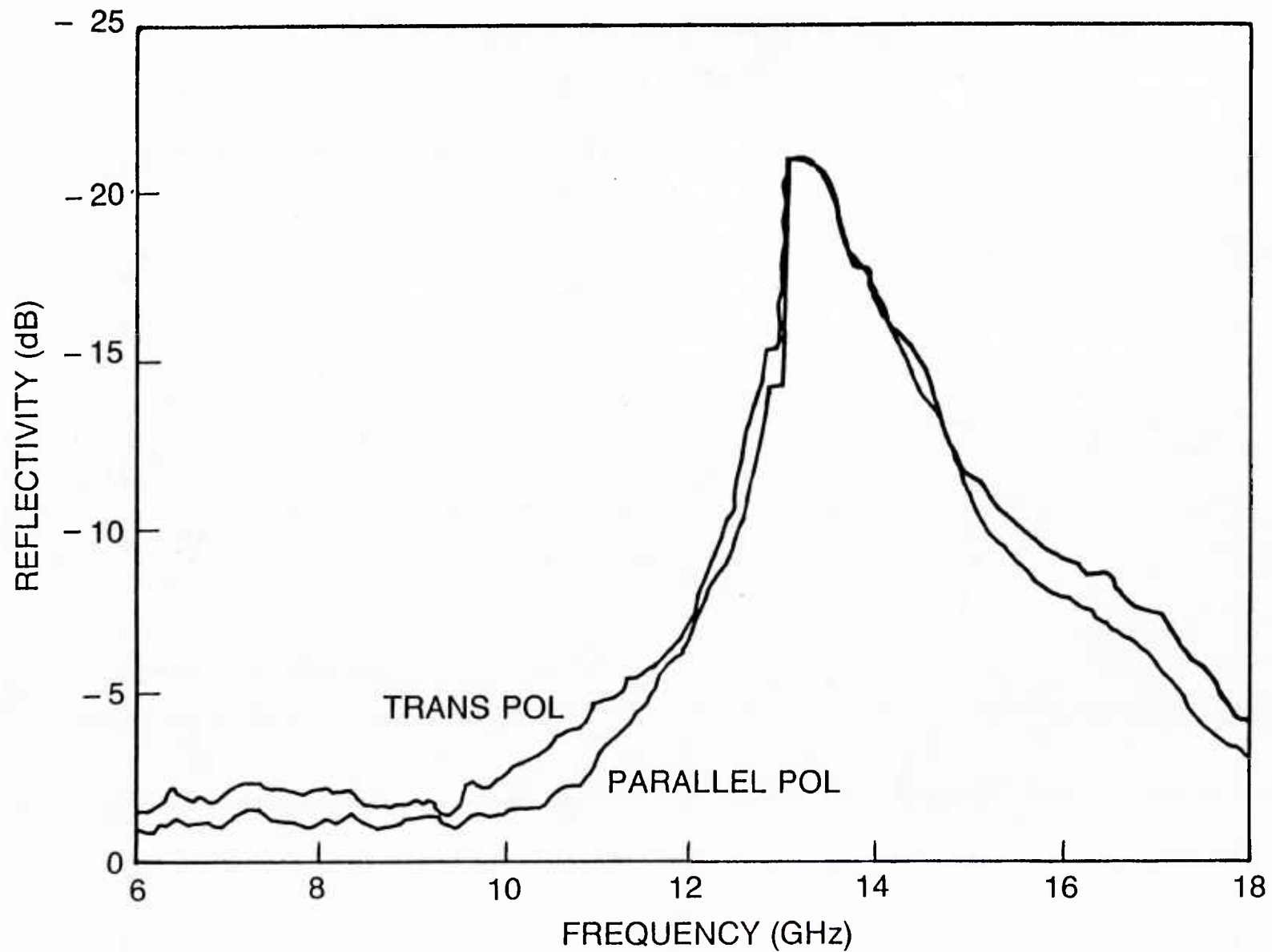


Fig. 6 — Reflectivity of epoxy panel at 6090 lb load

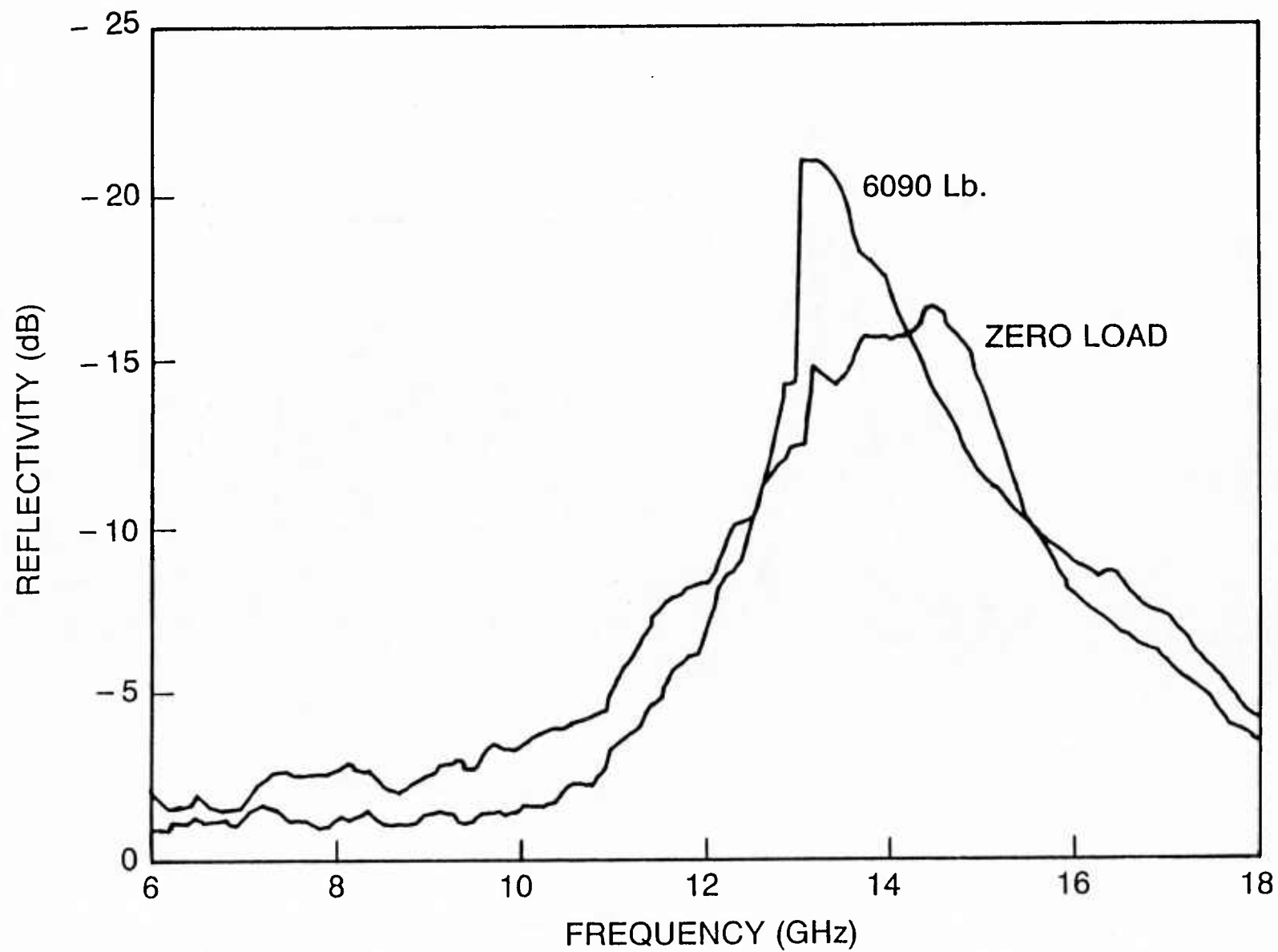


Fig. 7 — Reflectivity of epoxy panel with parallel polarization

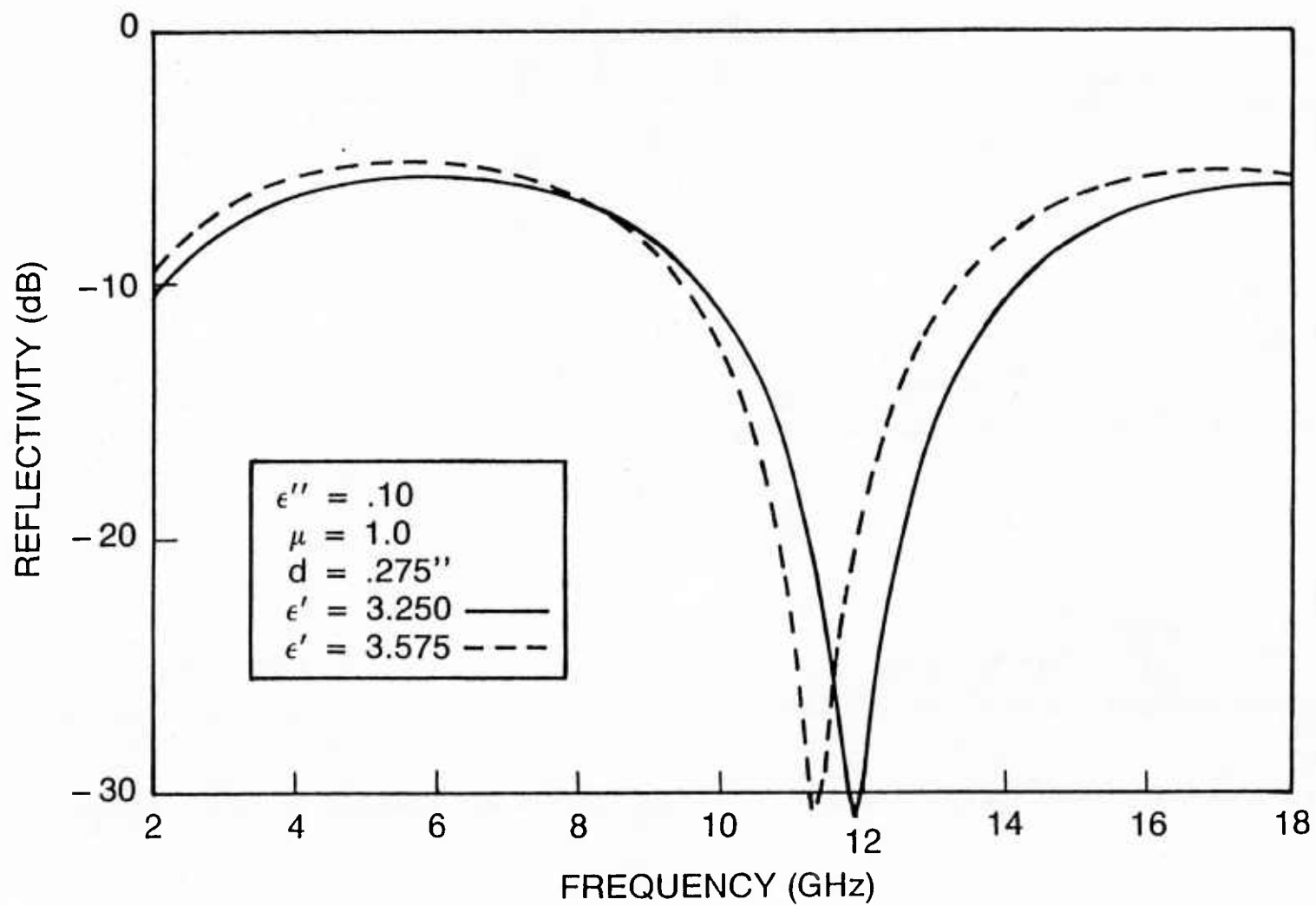


Fig. 8 — Effect of  $\epsilon'$  variability on reflectivity

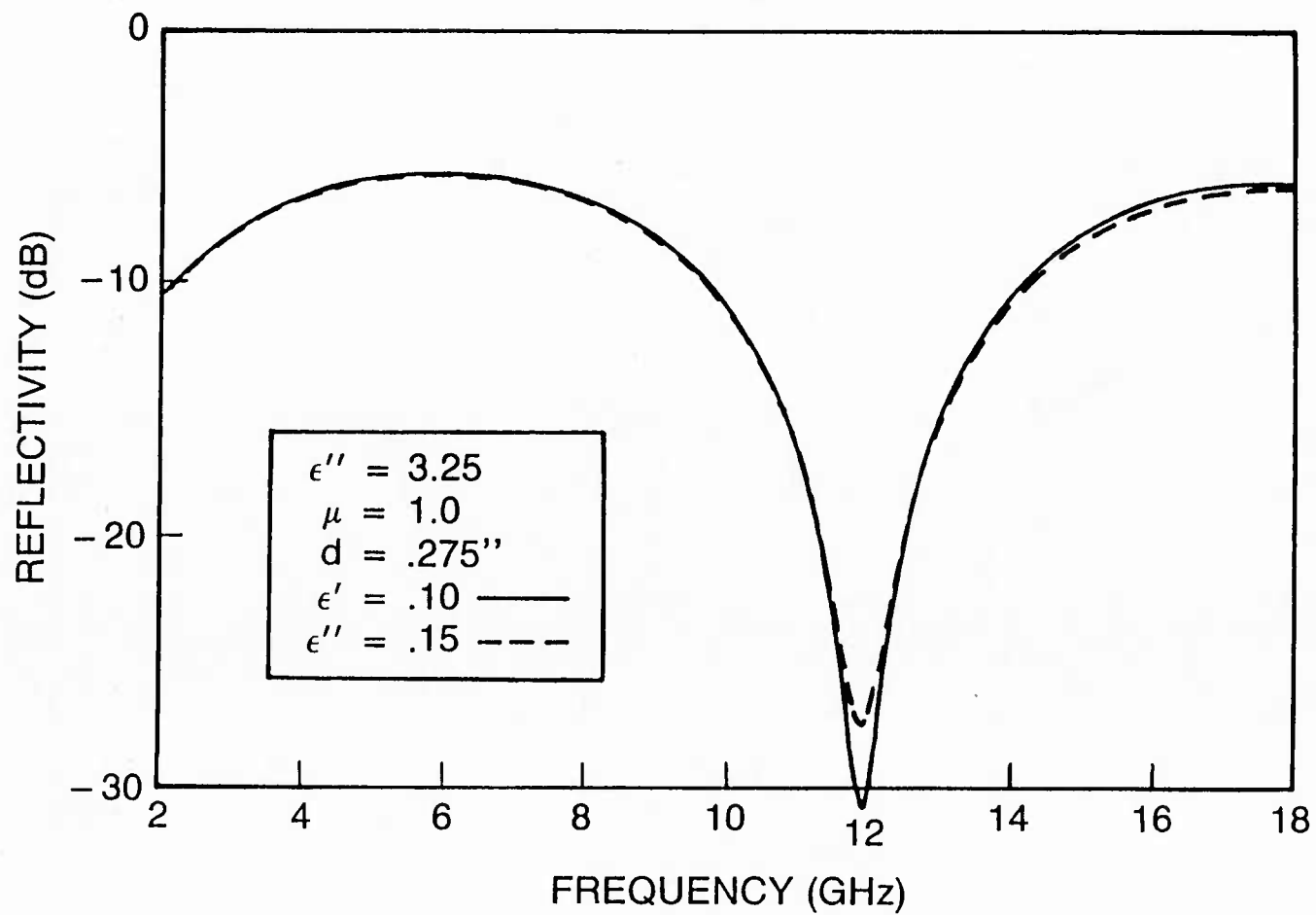


Fig. 9 — Effect of  $\epsilon''$  variability on reflectivity



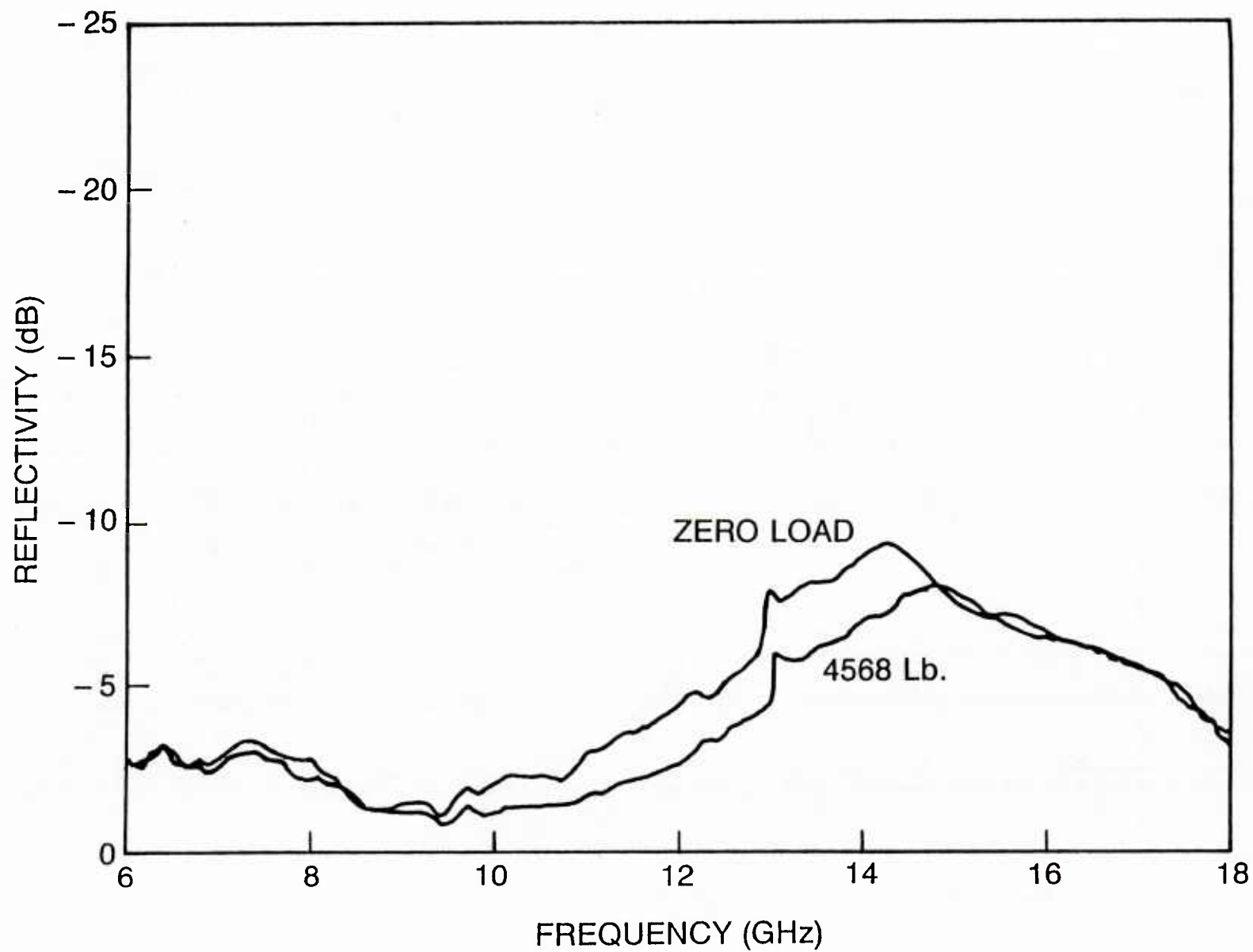


Fig. 10 — Reflectivity of hybrid panel for transverse polarization

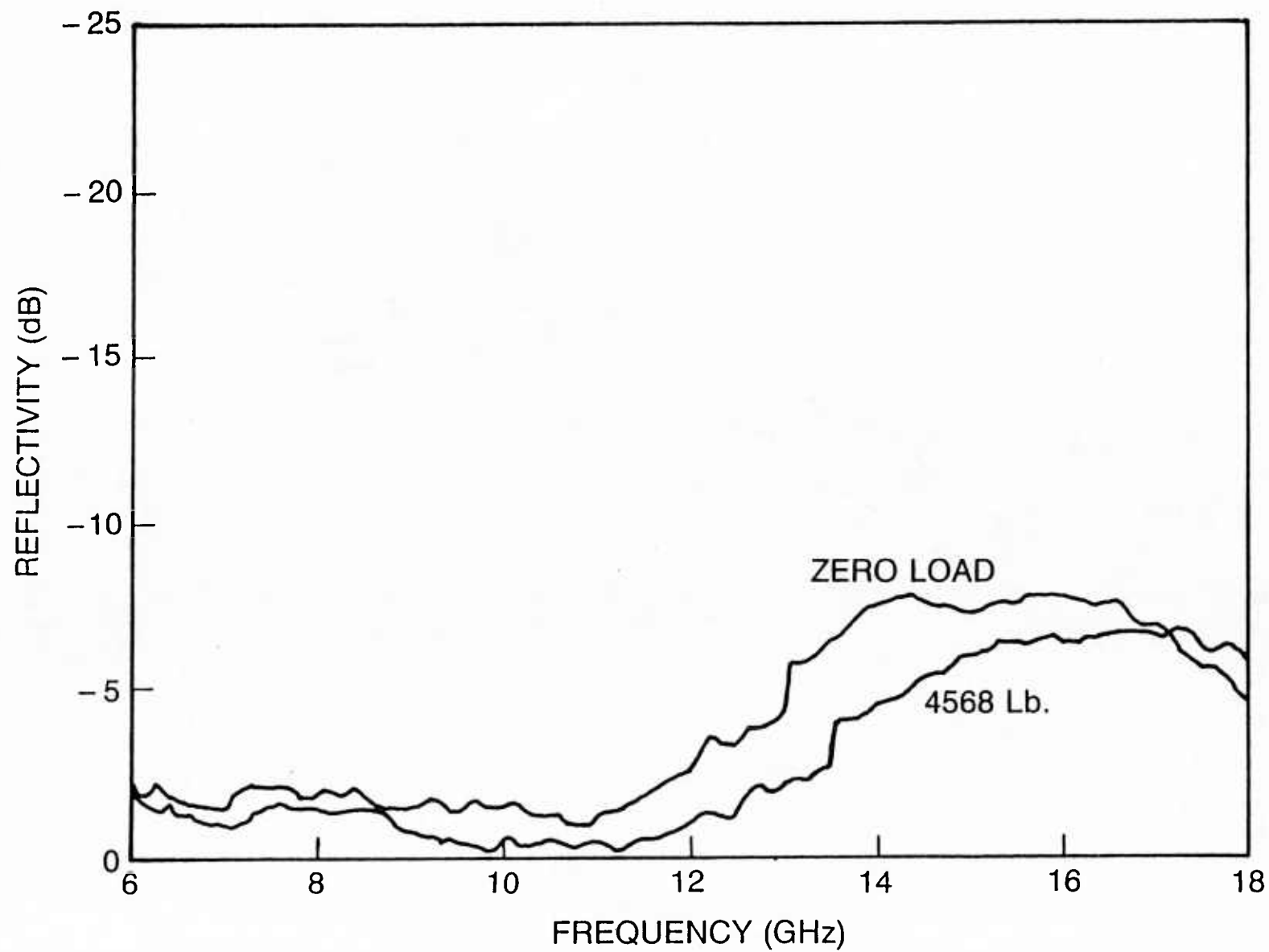


Fig. 11 — Reflectivity of hybrid panel for parallel polarization

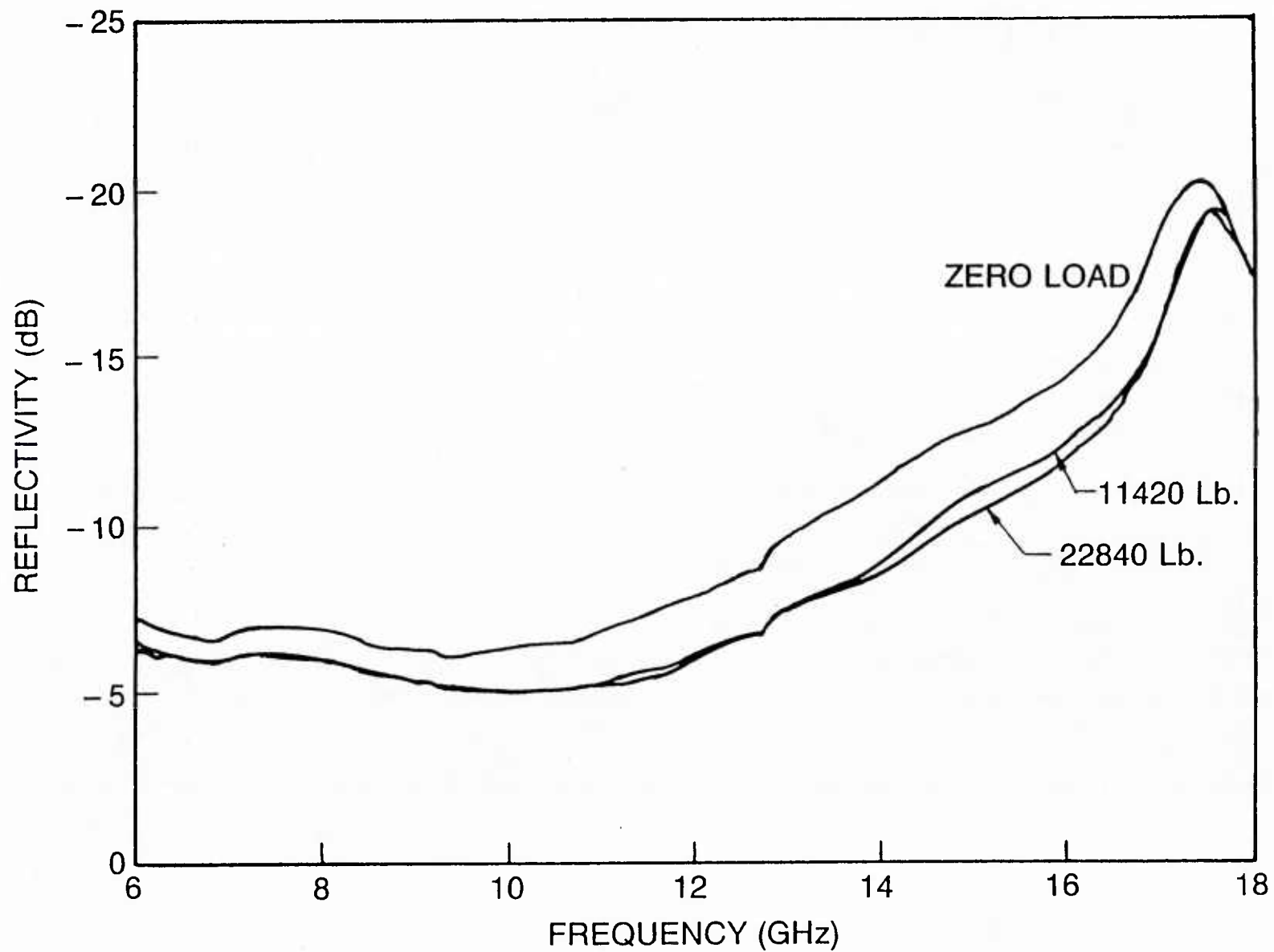


Fig. 12 — Reflectivity of fiberglass/epoxy panel for transverse polarization

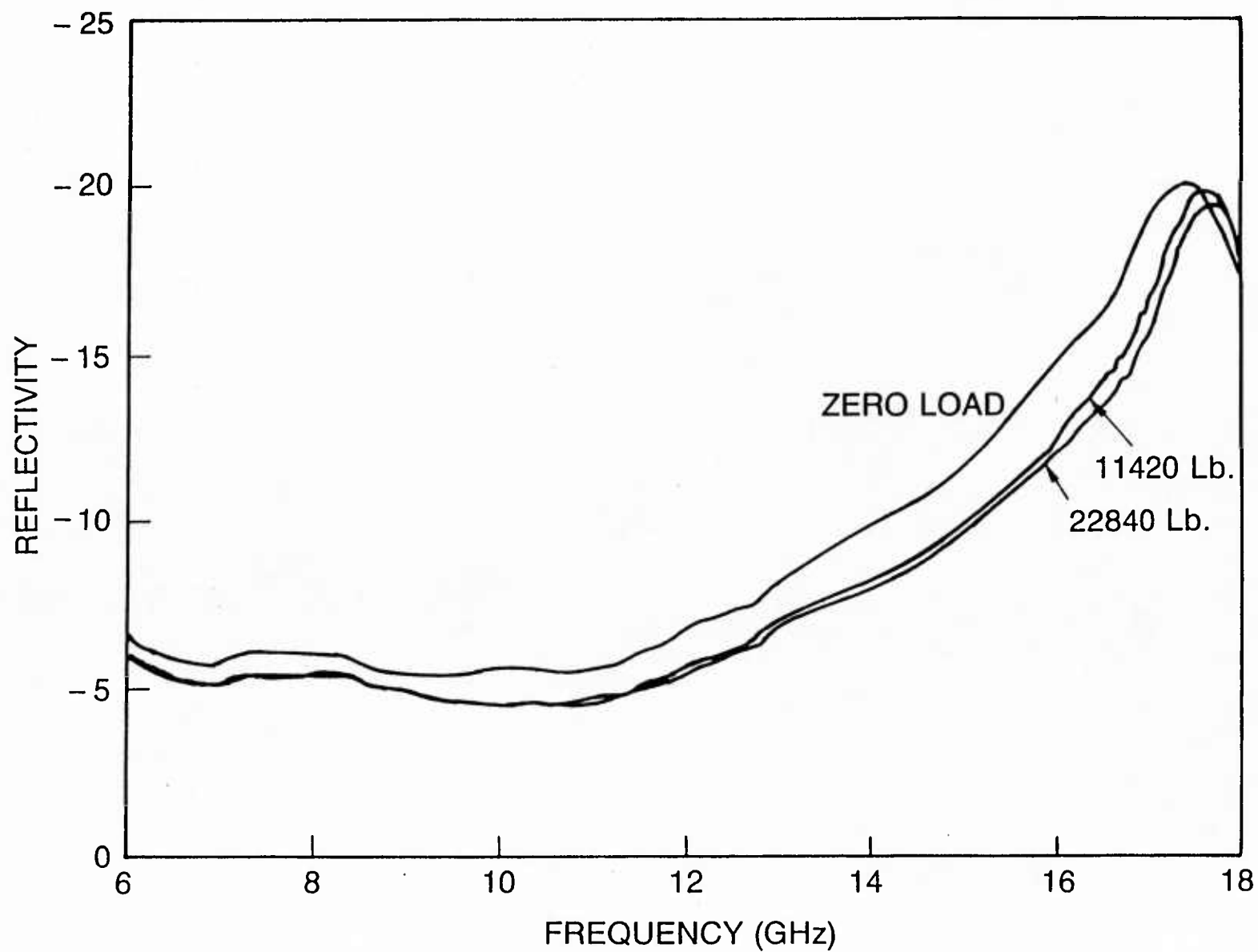


Fig. 13 — Reflectivity of fiberglass/epoxy panel for parallel polarization

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